a.) $\dfrac{36ab^2}{-12a^2b^2} = -\dfrac{3}{a}$

b.) $\dfrac{9n^3}{3n^9} = \dfrac{3}{n^6}$

c.) $\dfrac{-5x}{-5x^5} = \dfrac{1}{x^4}$

d.) $\dfrac{(xy)^2}{-x^3y} = \dfrac{\cancel{x^2}y^{\cancel{2}}}{-x^{\cancel{3}}\cancel{y}} = -\dfrac{y}{x}$

e.) $\dfrac{2ab^2}{(a^2b)^2} = \dfrac{2\cancel{a}\cancel{b^2}}{a^4\cancel{b^2}} = \dfrac{2}{a^3}$

f.) $\dfrac{-2ab^2}{a(-2b)^2} = \dfrac{-\cancel{2}\cancel{a}\cancel{b^2}}{\cancel{a}\cdot \cancel{4}\cancel{b^2}} = -\dfrac{1}{2}$

g.) $\dfrac{(2a^3)^4}{(4a^2)^3} = \dfrac{\cancel{16}a^{12}}{\cancel{64}\cancel{a^6}} = \dfrac{a^6}{4}$

h.) $\dfrac{-3ab}{6ab-9} = \dfrac{-\cancel{3}ab}{\cancel{3}(2ab-3)} = -\dfrac{ab}{2ab-3}$

i.) $\dfrac{3x^2-x}{1-3x} = \dfrac{x(3x-1)}{1-3x} = \dfrac{x\cancel{(3x-1)}}{-1\cancel{(3x-1)}} = -x$

j.) $\dfrac{2a^2-2a}{a^2-a^3} = \dfrac{2a(a-1)}{a^2(1-a)} = \dfrac{2\cancel{a}\cancel{(a-1)}}{-\cancel{a^2}\cancel{(a-1)}} = -\dfrac{2}{a}$

k.) $\dfrac{9b^2-4}{3b-2} = \dfrac{(3b+2)\cancel{(3b-2)}}{\cancel{(3b-2)}} = 3b+2$

l.) $\dfrac{x^2+y^2}{(x+y)^2} = \dfrac{x^2+y^2}{(x+y)^2}$!

m.) $\dfrac{4a^2-4}{(a+1)(a-1)} = \dfrac{4(a^2-1)}{(a+1)(a-1)} = \dfrac{4\cancel{(a+1)}\cancel{(a-1)}}{\cancel{(a+1)}\cancel{(a-1)}} = 4$

n.) $\dfrac{-(yz)^4}{y(-z)^4} = \dfrac{-y^{\cancel{4}}\cancel{z^4}}{\cancel{y}\cancel{z^4}} = -\dfrac{y^3}{1} = -y^3$

o.) $\dfrac{(3z)^2}{4}\cdot\dfrac{5}{-2z}\cdot\dfrac{-(2z)^2}{15} = \dfrac{\cancel{9}z^{\cancel{2}}}{\cancel{4}}\cdot\dfrac{\cancel{5}}{\cancel{-2z}}\cdot\dfrac{-\cancel{4}z^2}{\cancel{15}} = \dfrac{3z^3}{2}$

p.) $\dfrac{11+b}{b^2-121} = \dfrac{\cancel{(b+11)}}{\cancel{(b+11)}(b-11)} = \dfrac{1}{b-11}$

q.) $\dfrac{m^3-m^2n}{m^2n-mn^2} = \dfrac{\cancel{m^2}\cancel{(m-n)}}{\cancel{mn}\cancel{(m-n)}} = \dfrac{m}{n}$

r.) $\dfrac{x^2-1}{x^2-2x+1} = \dfrac{(x+1)\cancel{(x-1)}}{(x-1)\cancel{(x-1)}} = \dfrac{x+1}{x-1}$

s.) $\dfrac{1-x^2}{x-x^2} = \dfrac{(1+x)\cancel{(1-x)}}{x\cancel{(1-x)}} = \dfrac{1+x}{x}$

t.) $\dfrac{k^2-k-6}{k^2-5k+6} = \dfrac{\cancel{(k-3)}(k+2)}{\cancel{(k-3)}(k-2)} = \dfrac{k+2}{k-2}$

1. Kürze:

a.) $\frac{-(3y)^2}{(-2x)^4}\cdot\frac{4x^2}{-2y^3} = \frac{+9\cancel{y^2}}{\cancel{16x^4}}\cdot\frac{\cancel{4x^2}}{+2\cancel{y^3}} = \frac{9}{8x^2y}$

b.) $\frac{(-2a^2b^3)^4}{-(5a^4b^3)^2} = \frac{16\cancel{a^8}\cancel{b^{12}}}{-25\cancel{a^8}\cancel{b^6}} = -\frac{16b^6}{25}$

c.) $\frac{2-2x^2}{4x-4} = \frac{2(1-x)^2}{4(x-1)} = \frac{2(1+x)(1-x)}{4(x-1)}$

$= \frac{-\cancel{2}(1+x)\cancel{(-1+x)}}{\cancel{4}\cancel{(x-1)}} = -\frac{1+x}{2}$

d.) $\frac{b^2-6b+8}{4b}\cdot\frac{2b^2-2b}{b^2-3b+2} = \frac{\cancel{(b-2)}(b-4)}{\cancel{4b}}\cdot\frac{\cancel{2b}\cancel{(b-1)}}{\cancel{(b-2)}\cancel{(b-1)}}$

$= \frac{b-4}{2}$

2. Vereinfache:

a.) $\frac{x^2+1}{x-1}-\frac{(x+1)}{x-1} = \frac{x^2+1-(x+1)}{x-1} = \frac{x^2+1-x-1}{x-1}$

$= \frac{x^2-x}{x-1} = \frac{x\cancel{(x-1)}}{\cancel{(x-1)}} = x$

b.) $\frac{1}{x}+\frac{2}{x^2}-\frac{3}{x^3} = \frac{x^2}{x^3}+\frac{2x}{x^3}-\frac{3}{x^3} = \frac{x^2+2x-3}{x^3}$

$= \frac{(x+3)(x-1)}{x^3}$

c.) $\frac{7x}{x-4}-\frac{6x}{x-3} = \frac{7x(x-3)}{(x-4)(x-3)} - \frac{6x(x-4)}{(x-4)(x-3)}$

$= \frac{7x^2-21x-6x^2+24x}{(x-4)(x-3)} = \frac{x^2+3x}{(x-4)(x-3)} = \frac{x(x+3)}{(x-4)(x-3)}$

d.) $\frac{a}{a^2-1}-\frac{1}{2a-2} = \frac{a}{(a+1)(a-1)} - \frac{1}{2(a-1)} = \frac{2a}{2(a+1)(a-1)} - \frac{(a+1)}{2(a+1)(a-1)}$

$= \frac{2a-a-1}{2(a+1)(a-1)} = \frac{\cancel{(a-1)}}{2(a+1)\cancel{(a-1)}} = \frac{1}{2(a+1)}$

e.) $\frac{2}{1}-\frac{1+2a}{a-1}+\frac{2}{a} = \frac{2a(a-1)}{a(a-1)} - \frac{a(1+2a)}{a(a-1)} + \frac{2(a-1)}{a(a-1)}$

$= \frac{2a^2-2a-a-2a^2+2a-2}{a(a-1)} = \frac{-a-2}{a(a-1)}$

f.) $\frac{x}{x^2-1}+\frac{2}{x^2-2x-3} = \frac{x}{(x+1)(x-1)} + \frac{2}{(x-3)(x+1)}$

$= \frac{x(x-3)}{(x+1)(x-1)(x-3)} + \frac{2(x-1)}{(x+1)(x-1)(x-3)} = \frac{x^2-3x+2x-2}{(x+1)(x-1)(x-3)}$

$= \frac{x^2-x-2}{(x+1)(x-1)(x-3)} = \frac{(x-2)\cancel{(x+1)}}{\cancel{(x+1)}(x-1)(x-3)} = \frac{x-2}{(x-1)(x-3)}$

1. $$\frac{x^2+3}{x-1}-\frac{(x+1)^2}{x-1}=\frac{x^2+3-[(x+1)(x+1)]}{x-1}=\frac{x^2+3-[x^2+2x+1]}{x-1}=\frac{x^2+3-x^2-2x-1}{x-1}=\frac{-2x+2}{x-1}=\frac{-2\cancel{(x-1)}}{\cancel{(x-1)}}=-2$$

2. $$\frac{a}{a-2}-\frac{a^2+4}{a^2-4}=\frac{a}{a-2}-\frac{a^2+4}{(a+2)(a-2)}=\frac{a(a+2)}{(a+2)(a-2)}-\frac{[a^2+4]}{(a+2)(a-2)}=\frac{a^2+2a-a^2-4}{(a+2)(a-2)}=\frac{2a-4}{(a+2)(a-2)}=\frac{2\cancel{(a-2)}}{(a+2)\cancel{(a-2)}}=\frac{2}{a+2}$$

3. $$\frac{2x}{x+2}-\frac{16}{x^2-4}=\frac{2x}{x+2}-\frac{16}{(x+2)(x-2)}=\frac{2x(x-2)}{(x+2)(x-2)}-\frac{16}{(x+2)(x-2)}=\frac{2x^2-4x-16}{(x+2)(x-2)}=\frac{2(x^2-2x-8)}{(x+2)(x-2)}=\frac{2(x-4)\cancel{(x+2)}}{\cancel{(x+2)}(x-2)}=\frac{2(x-4)}{x-2}$$

4. $$\frac{a}{a-b}-\frac{b}{a+b}-1=\frac{a(a+b)}{(a+b)(a-b)}-\frac{b(a-b)}{(a+b)(a-b)}-\frac{[(a+b)(a-b)]}{(a+b)(a-b)}=\frac{\cancel{a^2}+\cancel{ab}-\cancel{ab}+b^2-\cancel{a^2}+b^2}{(a+b)(a-b)}=\frac{2b^2}{(a+b)(a-b)}$$

5. $$\frac{x+2}{x^2-2x}-\frac{8}{x^2-4}=\frac{x+2}{x(x-2)}-\frac{8}{(x+2)(x-2)}=\frac{(x+2)(x+2)}{x(x+2)(x-2)}-\frac{8x}{x(x+2)(x-2)}=\frac{x^2+4x+4-8x}{x(x+2)(x-2)}=\frac{x^2-4x+4}{x(x+2)(x-2)}=\frac{(x-2)\cancel{(x-2)}}{x(x+2)\cancel{(x-2)}}=\frac{x-2}{x(x+2)}$$

6. $$\frac{1}{a^2-9}-\frac{1}{a^2-3a}-\frac{1}{3a}=\frac{1}{(a+3)(a-3)}-\frac{1}{a(a-3)}-\frac{1}{3a}=\frac{3a}{3a(a+3)(a-3)}-\frac{[3(a+3)]}{3a(a+3)(a-3)}-\frac{[(a+3)(a-3)]}{3a(a+3)(a-3)}=\frac{\cancel{3a}-\cancel{3a}-\cancel{9}-a^2+\cancel{9}}{3a(a+3)(a-3)}=\frac{-\cancel{a^2}\,a}{3\cancel{a}(a+3)(a-3)}=-\frac{a}{3(a+3)(a-3)}$$

7. $$\frac{x+5}{x-4}-\frac{x-4}{x+5}-\frac{12x-21}{x^2+x-20}=\frac{(x+5)(x+5)}{(x+5)(x-4)}-\frac{[(x-4)(x-4)]}{(x+5)(x-4)}-\frac{[12x-21]}{(x+5)(x-4)}=\frac{\cancel{x^2}+10x+25-\cancel{x^2}+8x-16-12x+21}{(x+5)(x-4)}=\frac{6x+30}{(x+5)(x-4)}=\frac{6\cancel{(x+5)}}{\cancel{(x+5)}(x-4)}=\frac{6}{x-4}$$

8. $$\frac{1}{5a+10}-\frac{1}{2a^2-6a}+\frac{1}{a^2-a-6}=\frac{1}{5(a+2)}-\frac{1}{2a(a-3)}+\frac{1}{(a-3)(a+2)}=\frac{2a(a-3)}{10a(a+2)(a-3)}-\frac{5(a+2)}{10a(a+2)(a-3)}+\frac{10a}{10a(a+2)(a-3)}=\frac{2a^2-6a-5a-10+10a}{10a(a+2)(a-3)}$$
$$=\frac{2a^2-a-10}{10a(a+2)(a-3)}=\frac{(2a-5)\cancel{(a+2)}}{10a\cancel{(a+2)}(a-3)}=\frac{2a-5}{10a(a-3)}$$

1. $$\frac{x-y}{5}-\frac{(x+y)}{5}=\frac{\cancel{x}-y-\cancel{x}-y}{5}=\frac{-2y}{5}=\underline{\underline{-\frac{2y}{5}}}$$

2. $$\frac{x-y-z}{3}-\frac{(y-x-z)}{3}-\frac{(z-x-y)}{3}=\frac{x-y\cancel{-z}\cancel{-y}+x\cancel{+z}-z+x\cancel{+y}}{3}=\underline{\underline{\frac{3x-y-z}{3}}}$$

3. $$\frac{2}{3x}-\frac{3}{2x}+1=\frac{4}{6x}-\frac{9}{6x}+\frac{6x}{6x}=\frac{4-9+6x}{6x}=\underline{\underline{\frac{-5+6x}{6x}}}$$

4. $$\frac{1}{a}+\frac{1}{a^2}-\frac{1}{a^3}=\frac{a^2}{a^3}+\frac{a}{a^3}-\frac{1}{a^3}=\underline{\underline{\frac{a^2+a-1}{a^3}}}$$

5. $$\frac{2}{ax}+\frac{1}{x^2}-\frac{3}{2a}=\frac{4x}{2ax^2}+\frac{2a}{2ax^2}-\frac{3x^2}{2ax^2}=\underline{\underline{\frac{4x+2a-3x^2}{2ax^2}}}$$

6. $$\frac{1}{2a-3}-\frac{1}{9-6a}-\frac{1}{4a-6}=\frac{1}{2a-3}-\frac{1}{3(3-2a)}-\frac{1}{2(2a-3)}=\frac{1}{2a-3}\oplus\frac{1}{3(2a-3)}-\frac{1}{2(2a-3)}=\frac{6}{6(2a-3)}\oplus\frac{2}{6(2a-3)}-\frac{3}{6(2a-3)}=\underline{\underline{\frac{5}{6(2a-3)}}}$$

7. $$\frac{a+1}{a-1}-\frac{a^2}{a^2-1}-\frac{1}{2a+2}=\frac{a+1}{a-1}-\frac{a^2}{(a+1)(a-1)}-\frac{1}{2(a+1)}=\frac{2(a+1)(a+1)}{2(a+1)(a-1)}-\frac{2a^2}{2(a+1)(a-1)}-\frac{(a-1)}{2(a+1)(a-1)}=\frac{\cancel{2a^2}+4a+2-\cancel{2a^2}-a+1}{2(a+1)(a-1)}=\frac{3a+3}{2(a+1)(a-1)}=\frac{3\overset{1}{\cancel{(a+1)}}}{2\underset{1}{\cancel{(a+1)}}(a-1)}=\underline{\underline{\frac{3}{2(a-1)}}}$$

8. $$\frac{2x^2+2y^2}{x^2-y^2}-\frac{x+y}{x-y}=\frac{2x^2+2y^2}{(x+y)(x-y)}-\frac{x+y}{x-y}=\frac{2x^2+2y^2}{(x+y)(x-y)}-\frac{[(x+y)(x+y)]}{(x+y)(x-y)}=\frac{2x^2+2y^2-x^2-2xy-y^2}{(x+y)(x-y)}=\frac{x^2-2xy+y^2}{(x+y)(x-y)}=\frac{(x-y)\overset{1}{\cancel{(x-y)}}}{(x+y)\underset{1}{\cancel{(x-y)}}}=\underline{\underline{\frac{x-y}{x+y}}}$$

9. $$\frac{z}{z-1}+\frac{1}{2z^2-2}-1=\frac{z}{z-1}+\frac{1}{2(z^2-1)}-\frac{1}{1}=\frac{z}{z-1}+\frac{1}{2(z+1)(z-1)}-\frac{1}{1}=\frac{2z(z+1)}{2(z+1)(z-1)}+\frac{1}{2(z+1)(z-1)}-\frac{[2(z+1)(z-1)]}{2(z+1)(z-1)}=\frac{\cancel{2z^2}+2z+1-\cancel{2z^2}+2}{2(z+1)(z-1)}=\underline{\underline{\frac{2z+3}{2(z+1)(z-1)}}}$$

10. $$\frac{a^2}{a^2-b^2}-\frac{b}{2a-2b}=\frac{a^2}{(a+b)(a-b)}-\frac{b}{2(a-b)}=\frac{2a^2}{2(a+b)(a-b)}-\frac{[b(a+b)]}{2(a+b)(a-b)}=\frac{2a^2-ab-b^2}{2(a+b)(a-b)}=\frac{(2a+b)\overset{1}{\cancel{(a-b)}}}{2(a+b)\underset{1}{\cancel{(a-b)}}}=\underline{\underline{\frac{2a+b}{2(a+b)}}}$$