1. a.) $p + \frac{9-p}{2} = \frac{2p}{2} + \frac{9-p}{2} = \frac{2p+9-p}{2} = \frac{p+9}{2}$

b.) $\frac{x-y}{3x} - 1 = \frac{x-y}{3x} - \frac{3x}{3x} = \frac{x-y-3x}{3x} = \frac{-2x-y}{3x}$

c.) $2 - \frac{k^2-k+1}{k^2} = \frac{2k^2-(k^2-k+1)}{k^2} = \frac{2k^2-k^2+k-1}{k^2} = \frac{k^2+k-1}{k^2}$

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2. a.) $\frac{2a}{a+b} + 1 = \frac{2a}{a+b} + \frac{a+b}{a+b} = \frac{2a+a+b}{a+b} = \frac{3a+b}{a+b}$

b.) $4 - \frac{u-v}{u+v} = \frac{4(u+v)}{u+v} - \frac{u-v}{u+v} = \frac{4u+4v-(u-v)}{u+v} =$

$\frac{4u+4v-u+v}{u+v} = \frac{3u+5v}{u+v}$

c.) $\frac{z^2}{z+1} - z = \frac{z^2}{z+1} - \frac{z(z+1)}{z+1} = \frac{z^2-z(z+1)}{z+1} =$

$\frac{z^2-z^2-z}{z+1} = -\frac{z}{z+1}$

---

3. a.) $3 - \frac{m}{m-n} = \frac{3(m-n)}{m-n} - \frac{m}{m-n} = \frac{3m-3n-m}{m-n} = \frac{2m-3n}{m-n}$

b.) $\frac{q}{q+1} - 1 = \frac{q}{q+1} - \frac{q+1}{q+1} = \frac{q-(q+1)}{q+1} = \frac{q-q-1}{q+1} = -\frac{1}{q+1}$

c.) $e - \frac{e^2-2}{e-2} = \frac{e(e-2)}{e-2} - \frac{e^2-2}{e-2} = \frac{e^2-2e-(e^2-2)}{e-2} =$

$\frac{e^2-2e-e^2+2}{e-2} = \frac{-2e+2}{e-2}$

d.) $1 + \frac{z}{1-z} = \frac{1-z}{1-z} + \frac{z}{1-z} = \frac{1-z+z}{1-z} = \frac{1}{1-z}$

$1 + z + \frac{z^2}{1-z} = \frac{1-z}{1-z} + \frac{z(1-z)}{1-z} + \frac{z^2}{1-z} =$

$\frac{1-z+z-z^2+z^2}{1-z} = \frac{1}{1-z}$

$1 + z + z^2 + \frac{z^3}{1-z} = \frac{1-z}{1-z} + \frac{z(1-z)}{1-z} + \frac{z^2(1-z)}{1-z} + \frac{z^3}{1-z} =$

$\frac{1-z+z-z^2+z^2-z^3+z^3}{1-z} = \frac{1}{1-z}$

4.

a.) $$\frac{1}{a+b}+\frac{1}{c}=\frac{c}{(a+b)\cdot c}+\frac{a+b}{(a+b)\cdot c}=\underline{\underline{\frac{c+a+b}{(a+b)\cdot c}}}$$

b.) $$\frac{8}{n+5}-\frac{n+2}{n}=\frac{8\cdot n}{(n+5)\cdot n}-\frac{(n+2)(n+5)}{(n+5)\cdot n}=$$
$$\frac{8n-[(n+2)(n+5)]}{(n+5)n}=\frac{8n-[n^2+7n+10]}{(n+5)n}=\frac{8n-n^2-7n-10}{(n+5)n}=$$
$$\underline{\underline{\frac{n-n^2-10}{(n+5)n}}}$$

c.) $$\frac{x+y}{x-y}-\frac{x-y}{x+y}=\frac{(x+y)(x+y)}{(x-y)(x+y)}-\frac{(x-y)(x-y)}{(x-y)(x+y)}=$$
$$\frac{x^2+2xy+y^2-[x^2-2xy+y^2]}{(x-y)(x+y)}=\frac{x^2+2xy+y^2-x^2+2xy-y^2}{(x-y)(x+y)}=$$
$$\underline{\underline{\frac{4xy}{(x-y)(x+y)}}}$$

5.

a.) $$\frac{m}{m-1}-\frac{m-1}{m+2}=\frac{m(m+2)}{(m-1)(m+2)}-\frac{(m-1)(m-1)}{(m-1)(m+2)}=$$
$$\frac{m^2+2m-[m^2-2m+1]}{(m-1)(m+2)}=\frac{m^2+2m-m^2+2m-1}{(m-1)(m+2)}=\underline{\underline{\frac{4m-1}{(m-1)(m+2)}}}$$

b.) $$\frac{2r}{s}-\frac{r+3}{r+s+1}=\frac{2r(r+s+1)}{s(r+s+1)}-\frac{s(r+3)}{s(r+s+1)}=$$
$$\frac{2r^2+2rs+2r-sr-3s}{s(r+s+1)}=\underline{\underline{\frac{2r^2+rs+2r-3s}{s(r+s+1)}}}$$

c.) $$\frac{w-4}{w-2}+\frac{w+6}{w+3}=\frac{(w-4)(w+3)}{(w-2)(w+3)}+\frac{(w+6)(w-2)}{(w-2)(w+3)}=$$
$$\frac{w^2-w-12+w^2+4w-12}{(w-2)(w+3)}=\underline{\underline{\frac{2w^2+3w-24}{(w-2)(w+3)}}}$$

6.

a.) $$\frac{c}{c+d}-\frac{c-d}{2(c+d)}=\frac{2c}{2(c+d)}-\frac{c-d}{2(c+d)}=\frac{2c-(c-d)}{2(c+d)}=$$
$$\frac{2c-c+d}{2(c+d)}=\frac{\overset{1}{\cancel{(c+d)}}}{2\underset{1}{\cancel{(c+d)}}}=\underline{\underline{\frac{1}{2}}}$$

b.) $$\frac{4}{z-1}+\frac{z}{z^2-1}=\frac{4}{z-1}+\frac{z}{(z+1)(z-1)}=\frac{4(z+1)}{(z+1)(z-1)}+\frac{z}{(z+1)(z-1)}=$$
$$\frac{4z+4+z}{(z+1)(z-1)}=\underline{\underline{\frac{5z+4}{(z+1)(z-1)}}}$$

c.) $$\frac{3u}{u^2+2uv+v^2}-\frac{1}{u+v}=\frac{3u}{(u+v)(u+v)}-\frac{1\cdot(u+v)}{(u+v)(u+v)}=$$
$$\frac{3u-u-v}{(u+v)(u+v)}=\underline{\underline{\frac{2u-v}{(u+v)(u-v)}}}$$

d.) $$\frac{a+2b+t}{4at+8bt}-\frac{1}{4t}=\frac{a+2b+t}{4t\cdot(a+2b)}-\frac{1\cdot(a+2b)}{4t(a+2b)}=$$
$$\frac{a+2b+t-a-2b}{4t\cdot(a+2b)}=\frac{\cancel{t}^{1}}{\cancel{4t}_{1}\cdot(a+2b)}=\underline{\underline{\frac{1}{4(a+2b)}}}$$

---

7. a.) $$\frac{x-y}{15x+10y}+\frac{x+y}{3x+2y}=\frac{x-y}{5(3x+2y)}+\frac{5(x+y)}{5(3x+2y)}=$$
$$\frac{x-y+5x+5y}{5(3x+2y)}=\frac{6x+4y}{5(3x+2y)}=\frac{2\cancel{(3x+2y)}^{1}}{5\cancel{(3x+2y)}_{1}}=\underline{\underline{\frac{2}{5}}}$$

b.) $$\frac{8p}{4p^2-4p+1}-\frac{3}{2p-1}=\frac{8p}{(2p-1)(2p-1)}-\frac{3(2p-1)}{(2p-1)(2p-1)}=$$
$$\frac{8p-6p+3}{(2p-1)(2p-1)}=\underline{\underline{\frac{2p+3}{(2p-1)(2p-1)}}}$$

c.) $$\frac{r+2}{5r^2}-\frac{4r+4}{5r^3+10r^2}=\frac{r+2}{5r^2}-\frac{4r+4}{5r^2(r+2)}=$$
$$\frac{(r+2)(r+2)}{5r^2(r+2)}-\frac{(4r+4)}{5r^2(r+2)}=\frac{r^2+4r+4-4r-4}{5r^2(r+2)}=$$
$$\frac{\cancel{r^2}^{1}}{5\cancel{r^2}_{1}(r+2)}=\underline{\underline{\frac{1}{5(r+2)}}}$$

d.) $$\frac{1}{q-1}-\frac{q^2+2}{q^3-1}=\frac{q^3-1}{(q-1)(q^3-1)}-\frac{(q^2+2)(q-1)}{(q-1)(q^3-1)}=$$
$$\frac{q^3-1-[q^3-q^2+2q-2]}{(q-1)(q^3-1)}=\frac{q^3-1-q^3+q^2-2q+2}{(q-1)(q^3-1)}=\frac{q^2-2q+1}{(q-1)(q^3-1)}=$$
$$\frac{\cancel{(q-1)}^{1}(q-1)}{\cancel{(q-1)}_{1}(q^3-1)}=\frac{q-1}{q^3-1}\left(=\frac{\cancel{(q-1)}^{1}}{\cancel{(q-1)}_{1}(q^2+q+1)}=\underline{\underline{\frac{1}{q^2+q+1}}}\right)$$

---

8. a.) $$\frac{c}{c-d}-\frac{2cd}{c^2-d^2}-\frac{d}{c+d}=\frac{c}{c-d}-\frac{2cd}{(c+d)(c-d)}-\frac{d}{c+d}=$$
$$\frac{c\cdot(c+d)}{(c+d)(c-d)}-\frac{2cd}{(c+d)(c-d)}-\frac{d(c-d)}{(c+d)(c-d)}=\frac{c^2+cd-2cd-cd+d^2}{(c+d)(c-d)}=$$
$$\frac{c^2-2cd+d^2}{(c+d)(c-d)}=\frac{(c-d)\cancel{(c-d)}^{1}}{(c+d)\cancel{(c-d)}_{1}}=\underline{\underline{\frac{c-d}{c+d}}}$$

b.) $$\frac{1}{a-2}+\frac{1}{a+5}-\frac{2a+3}{a^2+3a-10}=\frac{1}{a-2}+\frac{1}{a+5}-\frac{2a+3}{(a+5)(a-2)}=$$
$$\frac{1\cdot(a+5)}{(a+5)(a-2)}+\frac{1\cdot(a-2)}{(a+5)(a-2)}-\frac{(2a+3)}{(a+5)(a-2)}=\frac{a+5+a-2-2a-3}{(a+5)(a-2)}=$$
$$\frac{0}{(a+5)\cdot(a-2)}=\underline{\underline{0}}$$

9. a.)

$$\frac{z}{z-5}-\frac{5}{z+3}-\frac{40}{z^2-2z-15}=\frac{z}{z-5}-\frac{5}{z+3}-\frac{40}{(z+3)(z-5)}=$$

$$\frac{z\cdot(z+3)}{(z+3)(z-5)}-\frac{5\cdot(z-5)}{(z+3)(z-5)}-\frac{40}{(z+3)(z-5)}=\frac{z^2+3z-5z+25-40}{(z+3)(z-5)}=$$

$$\frac{z^2-2z-15}{(z+3)(z-5)}=\frac{\overset{1}{\cancel{(z+3)}}\,\overset{1}{\cancel{(z-5)}}}{\underset{1}{\cancel{(z+3)}}\,\underset{1}{\cancel{(z-5)}}}=\underline{\underline{1}}$$

b.)

$$\frac{n}{n+1}-\frac{2n+1}{n-1}+\frac{n^2+5n}{n^2-1}=\frac{n}{n+1}-\frac{2n+1}{n-1}+\frac{n^2+5n}{(n+1)(n-1)}=$$

$$\frac{n(n-1)}{(n+1)(n-1)}-\frac{[(2n+1)(n+1)]}{(n+1)(n-1)}+\frac{n^2+5n}{(n+1)(n-1)}=\frac{n^2-n-[2n^2+3n+1]+n^2+5n}{(n+1)(n-1)}=$$

$$\frac{n^2-n-2n^2-3n-1+n^2+5n}{(n+1)\cdot(n-1)}=\frac{\overset{1}{\cancel{(n-1)}}}{(n+1)\underset{1}{\cancel{(n-1)}}}=\underline{\underline{\frac{1}{n+1}}}$$

---

10. a.)

$$\frac{a-b}{4a+4b}+\frac{a+4b}{6a+6b}=\frac{a-b}{4(a+b)}+\frac{a+4b}{6(a+b)}=\frac{3(a-b)}{12(a+b)}+\frac{2(a+4b)}{12(a+b)}=$$

$$\frac{3a-3b+2a+8b}{12(a+b)}=\frac{5a+5b}{12(a+b)}=\frac{5\overset{1}{\cancel{(a+b)}}}{12\underset{1}{\cancel{(a+b)}}}=\underline{\underline{\frac{5}{12}}}$$

b.)

$$\frac{t+7}{3t-6}-\frac{t+4}{t^2-2t}=\frac{t+7}{3(t-2)}-\frac{t+4}{t(t-2)}=\frac{t(t+7)}{3t(t-2)}-\frac{3(t+4)}{3t(t-2)}=$$

$$\frac{t^2+7t-3t-12}{3t(t-2)}=\frac{t^2+4t-12}{3t(t-2)}=\frac{(t+6)\overset{1}{\cancel{(t-2)}}}{3t\underset{1}{\cancel{(t-2)}}}=\underline{\underline{\frac{t+6}{3t}}}$$

c.)

$$\frac{u}{uv+v^2}-\frac{v}{u^2+uv}=\frac{u}{v(u+v)}-\frac{v}{u(u+v)}=\frac{u^2}{uv(u+v)}-\frac{v^2}{uv(u+v)}=$$

$$\frac{u^2-v^2}{uv(u+v)}=\frac{\overset{1}{\cancel{(u+v)}}(u-v)}{uv\underset{1}{\cancel{(u+v)}}}=\underline{\underline{\frac{u-v}{uv}}}$$

d.)

$$\frac{c}{c^2-8c+16}+\frac{2}{c^2-6c+8}=\frac{c}{(c-4)(c-4)}+\frac{2}{(c-4)(c-2)}=$$

$$\frac{c(c-2)}{(c-4)(c-4)(c-2)}+\frac{2(c-4)}{(c-4)(c-4)(c-2)}=\frac{c^2-2c+2c-8}{(c-4)(c-4)(c-2)}=$$

$$\underline{\underline{\frac{c^2-8}{(c-4)(c-4)(c-2)}}}$$

---

11. a.)

$$\frac{1}{rx+ry}+\frac{1}{sx+sy}=\frac{1}{r(x+y)}+\frac{1}{s(x+y)}=$$

$$\frac{s}{rs(x+y)}+\frac{r}{rs(x+y)}=\underline{\underline{\frac{s+r}{rs(x+y)}}}$$

b.)

$$\frac{a}{a^2-b^2}+\frac{b}{(a-b)^2}=\frac{a}{(a+b)(a-b)}+\frac{b}{(a-b)(a-b)}=$$

$$\frac{a\cdot(a-b)}{(a+b)(a-b)(a-b)}+\frac{b(a+b)}{(a+b)(a-b)(a-b)}=\frac{a^2-ab+ab+b^2}{(a+b)(a-b)(a-b)}=$$

$$\underline{\underline{\frac{a^2+b^2}{(a+b)(a-b)(a-b)}}}$$

c.)

$$\frac{z+9}{z^2-1}-\frac{z+5}{z^2+z}=\frac{z+9}{(z+1)(z-1)}-\frac{z+5}{z(z+1)}=$$

$$\frac{z(z+9)}{z(z+1)(z-1)}-\frac{[(z+5)(z-1)]}{z(z+1)(z-1)}=\frac{z^2+9z-[z^2+4z-5]}{z(z+1)(z-1)}=$$

$$\frac{z^2+9z-z^2-4z+5}{z(z+1)(z-1)}=\frac{5z+5}{z(z+1)(z-1)}=\frac{5\cdot\cancel{(z+1)}^{1}}{z\cancel{(z+1)}_{1}(z-1)}=\underline{\underline{\frac{5}{z(z-1)}}}$$

d.)

$$\frac{5}{n^2+n-6}-\frac{3}{n^2-n-2}=\frac{5}{(n+3)(n-2)}-\frac{3}{(n-2)(n+1)}=$$

$$\frac{5(n+1)}{(n+3)(n-2)(n+1)}-\frac{3(n+3)}{(n+3)(n-2)(n+1)}=\frac{5n+5-3n-9}{(n+3)(n-2)(n+1)}=$$

$$\frac{2n-4}{(n+3)(n-2)(n+1)}=\frac{2\cancel{(n-2)}^{1}}{(n+3)\cancel{(n-2)}_{1}(n+1)}=\underline{\underline{\frac{2}{(n+3)(n+1)}}}$$

---

12. a.)

$$\frac{7}{e-1}+\frac{6}{1-e}=\frac{7}{e-1}\ominus\frac{6}{e-1}=\frac{7-6}{e-1}=\underline{\underline{\frac{1}{e-1}}}$$

b.)

$$\frac{5}{3h-3}-\frac{4}{2-2h}=\frac{5}{3(h-1)}-\frac{4}{2(1-h)}=\frac{5}{3(h-1)}\oplus\frac{4}{2\cdot(h-1)}=$$

$$\frac{10}{6(h-1)}+\frac{12}{6(h-1)}=\frac{\cancel{22}^{11}}{{}_{3}\cancel{6}(h-1)}=\underline{\underline{\frac{11}{3(h-1)}}}$$

c.)

$$\frac{r-4}{5r+5}+\frac{2}{1-r^2}=\frac{r-4}{5(r+1)}+\frac{2}{(1+r)(1-r)}=$$

$$\frac{r-4}{5(r+1)}\ominus\frac{2}{(1+r)(r-1)}=\frac{(r-4)(r-1)}{5(r+1)(r-1)}-\frac{10}{5(r+1)(r-1)}=\frac{r^2-5r+4-10}{5(r+1)(r-1)}=$$

$$\frac{r^2-5r-6}{5(r+1)(r-1)}=\frac{(r-6)\cancel{(r+1)}^{1}}{5\cancel{(r+1)}_{1}(r-1)}=\underline{\underline{\frac{r-6}{5(r-1)}}}$$

d.) $$\frac{u}{u-v}-\frac{4uv}{u^2-v^2}-\frac{v}{v-u}=\frac{u}{u-v}-\frac{4uv}{(u+v)(u-v)}+\frac{v}{u-v}=$$

$$\frac{u(u+v)}{(u+v)(u-v)}-\frac{4uv}{(u+v)(u-v)}+\frac{v(u+v)}{(u+v)(u-v)}=\frac{u^2+uv-4uv+uv+v^2}{(u+v)(u-v)}=$$

$$\frac{u^2-2uv+v^2}{(u+v)(u-v)}=\frac{(u-v)\cancel{(u-v)}^{1}}{(u+v)\cancel{(u-v)}_{1}}=\underline{\underline{\frac{u-v}{u+v}}}$$

---

13. a.) $$\frac{a-b}{c-d}-\frac{a+b}{d-c}=\frac{a-b}{c-d}+\frac{a+b}{c-d}=\frac{a-b+a+b}{c-d}=\underline{\underline{\frac{2a}{c-d}}}$$

b.) $$\frac{x+y}{2x-6y}+\frac{x+3y}{9y-3x}=\frac{x+y}{2(x-3y)}+\frac{x+3y}{3(3y-x)}=\frac{x+y}{2(x-3y)}-\frac{x+3y}{3(x-3y)}=$$

$$\frac{3(x+y)}{6(x-3y)}-\frac{2(x+3y)}{6(x-3y)}=\frac{3x+3y-2x-6y}{6(x-3y)}=\frac{\cancel{(x-3y)}^{1}}{6\cancel{(x-3y)}_{1}}=\underline{\underline{\frac{1}{6}}}$$

c.) $$\frac{8s}{s^2-4}+\frac{2+s}{2-s}=\frac{8s}{(s+2)(s-2)}-\frac{2+s}{s-2}=\frac{8s}{(s+2)(s-2)}-\frac{[(2+s)(s+2)]}{(s+2)(s-2)}=$$

$$\frac{8s-[s^2+4s+4]}{(s+2)(s-2)}=\frac{8s-s^2-4s-4}{(s+2)(s-2)}=\frac{-s^2+4s-4}{(s+2)(s-2)}=\frac{\cancel{(s-2)}^{1}(-s+2)}{(s+2)\cancel{(s-2)}_{1}}=\underline{\underline{\frac{-s+2}{s+2}}}$$

d.) $$\frac{m^2-8m}{2m^2+m-15}-\frac{m}{5-2m}=\frac{m^2-8m}{(2m-5)(m+3)}+\frac{m}{2m-5}=$$

$$\frac{m^2-8m}{(2m-5)(m+3)}+\frac{m(m+3)}{(2m-5)(m+3)}=\frac{m^2-8m+m^2+3m}{(2m-5)(m+3)}=\frac{2m^2-5m}{(2m-5)(m+3)}=$$

$$\frac{m\cancel{(2m-5)}^{1}}{\cancel{(2m-5)}_{1}(m+3)}=\underline{\underline{\frac{m}{m+3}}}$$

---

14. a.) $$\frac{2n-11}{3n-5}-\frac{4n+15}{n+7}+1=\frac{(2n-11)(n+7)}{(3n-5)(n+7)}-\frac{[(4n+15)(3n-5)]}{(3n-5)(n+7)}+\frac{(3n-5)(n+7)}{(3n-5)(n+7)}=$$

$$\frac{2n^2+3n-77-[12n^2+25n-75]+3n^2+16n-35}{(3n-5)(n+7)}=\frac{2n^2+3n-77-12n^2-25n+75+3n^2+16n-35}{(3n-5)(n+7)}=$$

$$\underline{\underline{\frac{-7n^2-6n-37}{(3n-5)(n+7)}}}$$

b.) $$\frac{2v+3w}{2v+w}-\frac{2v-w}{2v}-\frac{2v+3w}{w}=$$

$$\frac{2vw(2v+3w)}{2vw(2v+w)}-\frac{[w(2v+w)(2v-w)]}{2vw(2v+w)}-\frac{[2v(2v+w)(2v+3w)]}{2vw(2v+w)}=$$

%

$$\frac{4v^2w + 6vw^2 - [4v^2w - w^3] - [8v^2 + 16v^2w + 6vw^2]}{2vw\,(2v+w)} =$$

$$\frac{\cancel{4v^2w} + \cancel{6vw^2} - \cancel{4v^2w} + w^3 - 8v^2 - 16v^2w - \cancel{6vw^2}}{2vw\,(2v+w)} =$$

$$\underline{\underline{\frac{w^3 - 8v^2 - 16v^2w}{2vw\,(2v+w)}}}$$

---

15. a.) $$\frac{2r-19}{3r-7} - \frac{5r}{6r-8} - \frac{1}{2} = \frac{2r-19}{3r-7} - \frac{5r}{2(3r-4)} - \frac{1}{2} =$$

$$\frac{2(2r-19)(3r-4)}{2(3r-7)(3r-4)} - \frac{5r(3r-7)}{2(3r-7)(3r-4)} - \frac{[(3r-7)(3r-4)]}{2(3r-7)(3r-4)} =$$

$$\frac{12r^2 - 130r - 152 - 15r^2 + 35r - [9r^2 - 33r + 28]}{2(3r-7)(3r-4)} =$$

$$\frac{12r^2 - 130r - 152 - 15r^2 + 35r - 9r^2 + 33r - 28}{2(3r-7)(3r-4)} =$$

$$\frac{-12r^2 - 62r - 180}{2(3r-7)(3r-4)} = \frac{-\cancel{2}^{1}(6r^2 + 31r + 90)}{\cancel{2}_{1}(3r-7)(3r-4)} = \underline{\underline{-\frac{6r^2+31r+90}{(3r-7)(3r-4)}}}$$

b.) $$\frac{5}{p-2} - \frac{3}{2p+1} + \frac{1}{p+1} =$$

$$\frac{5(2p+1)(p+1)}{(p-2)(2p+1)(p+1)} - \frac{[3(p-2)(p+1)]}{(p-2)(2p+1)(p+1)} + \frac{(p-2)(2p+1)}{(p-2)(2p+1)(p+1)} =$$

$$\frac{10p^2 + 15p + 5 - [3p^2 - 3p - 6] + 2p^2 - 3p - 2}{(p-2)(2p+1)(p+1)} =$$

$$\frac{10p^2 + 15p + 5 - 3p^2 + 3p + 6 + 2p^2 - 3p - 2}{(p-2)(2p+1)(p+1)} =$$

$$\frac{9p^2 + 15p + 9}{(p-2)(2p+1)(p+1)} = \underline{\underline{\frac{3(3p^2+5p+3)}{(p-2)(2p+1)(p+1)}}}$$

---

16. a.)

$$\frac{5}{4x-8y} - \frac{3}{10y-5x} - \frac{11}{6x-12y} =$$

$$\frac{5}{4(x-2y)} - \frac{3}{5(2y-x)} - \frac{11}{6(x-2y)} =$$

$$\frac{5}{4(x-2y)} + \frac{3}{5\cdot(x-2y)} - \frac{11}{6(x-2y)} =$$

$$\frac{75}{60(x-2y)} + \frac{36}{60(x-2y)} - \frac{110}{60(x-2y)} =$$

$$\frac{75+36-110}{60(x-2y)} = \underline{\underline{\frac{1}{60(x-2y)}}}$$

b.)

$$\frac{b-c}{a^2+ac} - \frac{a-b}{ac+c^2} + \frac{a^2+c^2}{a^2c+ac^2} =$$

$$\frac{b-c}{a(a+c)} - \frac{a-b}{c(a+c)} + \frac{a^2+c^2}{ac(a+c)} =$$

$$\frac{c(b-c)}{ac(a+c)} - \frac{a(a-b)}{ac(a+c)} + \frac{a^2+c^2}{ac(a+c)} =$$

$$\frac{bc-\cancel{c^2}-\cancel{a^2}+ab+\cancel{a^2}+\cancel{c^2}}{ac(a+c)} = \frac{bc+ab}{ac(a+c)} = \frac{b\cdot\cancel{(c+a)}^{1}}{ac\cancel{(a+c)}_{1}} = \underline{\underline{\frac{b}{ac}}}$$

---

17. a.)

$$\frac{k+2}{6k-15} + \frac{8k+1}{8k-20} + \frac{k+11}{10-4k} = \frac{k+2}{3(2k-5)} + \frac{8k+1}{4(2k-5)} + \frac{k+11}{2(5-2k)} =$$

$$\frac{k+2}{3(2k-5)} + \frac{8k+1}{4(2k-5)} - \frac{k+11}{2(2k-5)} = \frac{4(k+2)}{12(2k-5)} + \frac{3(8k+1)}{12(2k-5)} - \frac{6(k+11)}{12(2k-5)} =$$

$$\frac{4k+8+24k+3-6k-66}{12(2k-5)} = \frac{22k-55}{12(2k-5)} = \frac{11\cancel{(2k-5)}^{1}}{12\cancel{(2k-5)}_{1}} = \underline{\underline{\frac{11}{12}}}$$

b.)

$$\frac{u}{u-v} + \frac{v}{v-u} - \frac{u+v-1}{u+v} = \frac{u}{u-v} - \frac{v}{u-v} - \frac{u+v-1}{u+v} =$$

$$\frac{u(u+v)}{(u+v)(u-v)} - \frac{v(u+v)}{(u+v)(u-v)} - \frac{[(u+v-1)(u-v)]}{(u+v)(u-v)} =$$

$$\frac{u^2+uv-uv-v^2-[u^2-v^2-u+v]}{(u+v)(u-v)} =$$

$$\frac{\cancel{u^2}+\cancel{uv}-\cancel{uv}-\cancel{v^2}-\cancel{u^2}+\cancel{v^2}+u-v}{(u+v)(u-v)} = \frac{\cancel{(u-v)}^{1}}{(u+v)\cancel{(u-v)}_{1}} = \underline{\underline{\frac{1}{u+v}}}$$

---

18. a)

$$\frac{2x-1}{x-3}-\frac{2x(x+2)}{x^2-9}-\frac{2}{3x}=\frac{2x-1}{x-3}-\frac{2x(x+2)}{(x+3)(x-3)}-\frac{2}{3x}=$$

$$\frac{3x(x+3)(2x-1)}{3x(x+3)(x-3)}-\frac{6x^2(x+2)}{3x(x+3)(x-3)}-\frac{[2(x+3)(x-3)]}{3x(x+3)(x-3)}=$$

$$\frac{6x^3+15x^2-9x-6x^3-12x^2-[2x^2-18]}{3x(x+3)(x-3)}=$$

$$\frac{\cancel{6x^3}+15x^2-9x-\cancel{6x^3}-12x^2-2x^2+18}{3x(x+3)(x-3)}=$$

$$\frac{x^2-9x+18}{3x(x+3)(x-3)}=\frac{\cancel{(x-3)}^{1}(x-6)}{3x(x+3)\cancel{(x-3)}_{1}}=\underline{\underline{\frac{x-6}{3x(x+3)}}}$$

b)

$$\frac{3s}{(s-2)^2}-\frac{2}{s}+\frac{s+4}{2s-s^2}=\frac{3s}{(s-2)(s-2)}-\frac{2}{s}+\frac{s+4}{s(2-s)}=$$

$$\frac{3s}{(s-2)(s-2)}-\frac{2}{s}\ominus\frac{s+4}{s(s-2)}=$$

$$\frac{3s^2}{s(s-2)(s-2)}-\frac{[2(s-2)(s-2)]}{s(s-2)(s-2)}-\frac{[(s+4)(s-2)]}{s(s-2)(s-2)}=$$

$$\frac{3s^2-[2s^2-8s+8]-[s^2+2s-8]}{s(s-2)(s-2)}=$$

$$\frac{3s^2-2s^2+8s\cancel{-8}-s^2-2s\cancel{+8}}{s(s-2)(s-2)}=\frac{6\cancel{s}^{1}}{\cancel{s}_{1}(s-2)(s-2)}=\underline{\underline{\frac{6}{(s-2)(s-2)}}}$$

---

19. a)

$$\frac{2u-v}{2u-2v}-\frac{u-v}{3u+3v}-\frac{v(3v-u)}{3v^2-3u^2}=$$

$$\frac{2u-v}{2(u-v)}-\frac{u-v}{3(u+v)}-\frac{v(3v-u)}{3(v^2-u^2)}=$$

$$\frac{2u-v}{2(u-v)}-\frac{u-v}{3(u+v)}\oplus\frac{v(3v-u)}{3(u^2-v^2)}=$$

$$\frac{2u-v}{2(u-v)}-\frac{u-v}{3(u+v)}+\frac{v(3v-u)}{3(u+v)(u-v)}=$$

$$\frac{3(u+v)(2u-v)}{6(u+v)(u-v)}-\frac{[2(u-v)(u-v)]}{6(u+v)(u-v)}+\frac{2v(3v-u)}{6(u+v)(u-v)}=$$

$$\frac{6u^2+3uv-3v^2-[2u^2-4uv+2v^2]+6v^2-2uv}{6(u+v)(u-v)}=$$

$$\frac{6u^2+3uv-3v^2-2u^2+4uv-2v^2+6v^2-2uv}{6(u+v)(u-v)}=\frac{4u^2+5uv+v^2}{6(u+v)(u-v)}=$$

$$\frac{(4u+r)\cancel{(u+r)}^{1}}{6\cancel{(u+r)}_{1}(u-r)} = \frac{4u+r}{6(u-r)}$$

b.) $$\frac{1}{z^2-z} - \frac{2}{z^2} + \frac{1}{z^2+z} = \frac{1}{z(z-1)} - \frac{2}{z^2} + \frac{1}{z(z+1)} =$$

$$\frac{z\cdot(z+1)}{z^2(z+1)(z-1)} - \frac{[2(z+1)(z-1)]}{z^2(z+1)(z-1)} + \frac{z(z-1)}{z^2(z+1)(z-1)} =$$

$$\frac{z^2+z-[2z^2-2]+z^2-z}{z^2(z+1)(z-1)} =$$

$$\frac{z^2+z-2z^2+2+z^2-z}{z^2(z+1)(z-1)} = \frac{2}{z^2(z+1)(z-1)}$$

---

20. $$\frac{a}{(a-b)(a-c)} + \frac{b}{(b-c)(b-a)} + \frac{c}{(c-a)(c-b)} =$$

$$\frac{a}{(a-b)(a-c)} - \frac{b}{(b-c)(a-b)} + \frac{c}{(a-c)(b-c)} =$$

$$\frac{a(b-c)}{(a-b)(a-c)(b-c)} - \frac{b(a-c)}{(a-b)(a-c)(b-c)} + \frac{c(a-b)}{(a-b)(a-c)(b-c)} =$$

$$\frac{\cancel{ab}-\cancel{ac}-\cancel{ab}+\cancel{bc}+\cancel{ac}-\cancel{bc}}{(a-b)(a-c)(b-c)} = 0$$

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21. $$\frac{x^4+36x^2-32}{x^4-8x^2+16} - \frac{16x}{x^3+2x^2-4x-8} - \frac{16x}{x^3-2x^2-4x+8} - 1 =$$

$$\frac{x^4+36x^2-32}{(x^2-4)(x^2-4)} - \frac{16x}{(x^2-4)(x+2)} - \frac{16x}{(x^2-4)(x-2)} - 1 =$$

$$\frac{x^4+36x^2-32}{(x^2-4)(x+2)(x-2)} - \frac{16x(x-2)}{(x^2-4)(x+2)(x-2)} - \frac{16x(x+2)}{(x^2-4)(x+2)(x-2)} - \frac{[(x^2-4)(x+2)(x-2)]}{(x^2-4)(x+2)(x-2)}$$

$$\frac{x^4+36x^2-32-16x^2+32x-16x^2-32x-[(x^2-4)(x^2-4)]}{(x^2-4)(x+2)(x-2)} =$$

$$\frac{x^4+36x^2-32-16x^2+32x-16x^2-32x-[x^4-8x^2+16]}{(x^2-4)(x+2)(x-2)} =$$

$$\frac{\cancel{x^4}+36x^2-32-16x^2\cancel{+32x}-16x^2\cancel{-32x}\cancel{-x^4}+8x^2-16}{(x^2-4)(x+2)(x-2)} =$$

$$\frac{12x^2-48}{(x^2-4)(x+2)(x-2)} = \frac{12\cancel{(x^2-4)}^{1}}{\cancel{(x^2-4)}_{1}(x+2)(x-2)} = \frac{12}{(x+2)(x-2)}$$

22.

$$\frac{a^2+3a+5}{a^4-a^3-31a^2+25a+150}-\frac{a+2}{a^3-3a^2-25a+75}+\frac{a-3}{a^3+2a^2-25a-50}-\frac{a-5}{a^3+4a^2-11a-30}=$$

$$\frac{a^2+3a+5}{(a^2-a-6)(a^2-25)}-\frac{a+2}{(a-3)(a^2-25)}+\frac{a-3}{(a+2)(a^2-25)}-\frac{a-5}{(a+5)(a^2-a-6)}=$$

$$\frac{a^2+3a+5}{(a-3)(a+2)(a+5)(a-5)}-\frac{a+2}{(a-3)(a+5)(a-5)}+\frac{a-3}{(a+2)(a+5)(a-5)}-\frac{a-5}{(a+5)(a-3)(a+2)}=$$

$$\frac{a^2+3a+5}{(a-3)(a+2)(a+5)(a-5)}-\frac{[(a+2)(a+2)]}{(a-3)(a+2)(a+5)(a-5)}+\frac{(a-3)(a-3)}{(a-3)(a+2)(a+5)(a-5)}-\frac{[(a-5)(a-5)]}{(a-3)(a+2)(a+5)(a-5)}=$$

$$\frac{a^2+3a+5-[a^2+4a+4]+a^2-6a+9-[a^2-10a+25]}{(a-3)(a+2)(a+5)(a-5)}=$$

$$\frac{\cancel{a^2}+3a+5-\cancel{a^2}-4a-4+\cancel{a^2}-6a+9-\cancel{a^2}+10a-25}{(a-3)(a+2)(a+5)(a-5)}=$$

$$\frac{3a-15}{(a-3)(a+2)(a+5)(a-5)}=\frac{3\cancel{(a-5)}^{1}}{(a-3)(a+2)(a+5)\cancel{(a-5)}_{1}}=\underline{\underline{\frac{3}{(a-3)(a+2)(a+5)}}}$$

23. $$\frac{6-x}{x^4+2x^3-13x^2-14x+24}+\frac{1}{x^3-2x^2-5x+6}+\frac{1}{x^2+x-2}-\frac{1}{x^2-4x+3}=$$

$$\frac{6-x}{(x^3-2x^2-5x+6)(x+4)}+\frac{1}{(x^2-4x+3)(x+2)}+\frac{1}{(x+2)(x-1)}-\frac{1}{(x-3)(x-1)}=$$

$$\frac{6-x}{(x^2-4x+3)(x+2)(x+4)}+\frac{1}{(x-3)(x-1)(x+2)}+\frac{1}{(x+2)(x-1)}-\frac{1}{(x-3)(x-1)}=$$

$$\frac{6-x}{(x-3)(x-1)(x+2)(x+4)}+\frac{1}{(x-3)(x-1)(x+2)}+\frac{1}{(x+2)(x-1)}-\frac{1}{(x-3)(x-1)}=$$

$$\frac{6-x}{(x-3)(x-1)(x+2)(x+4)}+\frac{x+4}{(x-3)(x-1)(x+2)(x+4)}+\frac{(x-3)(x+4)}{(x-3)(x-1)(x+2)(x+4)}-\frac{[(x+2)(x+4)]}{(x-3)(x-1)(x+2)(x+4)}=$$

$$\frac{6-x+x+4+x^2+x-12-[x^2+6x+8]}{(x-3)(x-1)(x+2)(x+4)}=$$

$$\frac{6\cancel{-x}\cancel{+x}+4\cancel{+x^2}+x-12\cancel{-x^2}-6x-8}{(x-3)(x-1)(x+2)(x+4)}=$$

$$\frac{-5x-10}{(x-3)(x-1)(x+2)(x+4)}=\frac{-5\cancel{(x+2)}^{1}}{(x-3)(x-1)\cancel{(x+2)}_{1}(x+4)}=-\frac{5}{(x-3)(x-1)(x+4)}$$