

Exponentialgleichungen (Lösungen)

1. $3^x = 2^{x+1} \quad | \text{Log}$

$$\log 3^x = \log 2^{x+1}$$
$$x \cdot \log 3 = (x+1) \cdot \log 2$$
$$x \cdot \log 3 = x \cdot \log 2 + \log 2 \quad | -x \cdot \log 2$$
$$x \cdot \log 3 - x \cdot \log 2 = \log 2$$
$$x \cdot (\log 3 - \log 2) = \log 2 \quad | : (\log 3 - \log 2)$$
$$x = \frac{\log 2}{\log 3 - \log 2}$$
$$\approx \underline{\underline{1,7}}$$

2. $6^x + 7^x = 6^{x+2} \quad | -6^x$

$$7^x = 6^{x+2} - 6^x$$
$$7^x = 6^x \cdot (6^2 - 1) \quad | : 6^x$$
$$\frac{7^x}{6^x} = 6^2 - 1$$
$$\left(\frac{7}{6}\right)^x = 35 \quad | \text{Log}$$
$$x \cdot \log \frac{7}{6} = \log 35 \quad | : \log \frac{7}{6}$$
$$x = \frac{\log 35}{\log \frac{7}{6}}$$
$$\approx \underline{\underline{23,1}}$$

3.

$$5^x + 5^{x+2} = 100$$

$$5^x \cdot (1 + 5^2) = 100 \quad | : (1 + 5^2)$$

$$5^x = \frac{100}{1+5^2} = \frac{100}{26} \quad | \log$$

$$x \cdot \log 5 = \log \frac{100}{26} \quad | : \log 5$$

$$x = \frac{\log \frac{100}{26}}{\log 5}$$

$$\hat{=} \underline{\underline{0,84}}$$

4.

$$4^x - 2^{x+4} = 60$$

$$(2^2)^x - 2^{x+4} = 60$$

$$2^{2 \cdot x} - 2^{x+4} = 60$$

$$2^x \cdot 2^x - 2^x \cdot 2^4 = 60$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$y \cdot y - y \cdot 2^4 = 60 \quad (, \text{Substitution})$$

$$y^2 - 16y = 60$$

$$(y-8) \cdot (y-8) = 124 \quad | \sqrt{\quad} \quad (, \text{Quadratisch erg\u00e4nzen})$$

$$y-8 = \sqrt{124} \quad | +8$$

$$y = \sqrt{124} + 8$$

$$\downarrow$$

$$2^x = \sqrt{124} + 8 \quad | \log \quad (, \text{R\u00fcck-Substitution})$$

$$x \cdot \log 2 = \log(\sqrt{124} + 8) \quad | : \log 2$$

$$x = \frac{\log(\sqrt{124} + 8)}{\log 2} \hat{=} \underline{\underline{4,258}}$$