Die Wurzel von 2 mit Hilfe eines Kettenbruches näherungsweise berechnen

($\sqrt{2} \cong 1{,}41421356...$)

$$\sqrt{2} = 1+\sqrt{2}-1 = 1+\cfrac{1}{\cfrac{1}{\sqrt{2}-1}} = 1+\cfrac{1}{\cfrac{\left(\sqrt{2}+1\right)}{\left(\sqrt{2}-1\right)\left(\sqrt{2}-1\right)}} = 1+\cfrac{1}{\cfrac{\left(\sqrt{2}+1\right)}{2-1}} = 1+\cfrac{1}{\sqrt{2}+1} = 1+\cfrac{1}{1+\sqrt{2}}$$

$\sqrt{2} = 1+\cfrac{1}{1+\sqrt{2}}$

fortlaufendes Einsetzen des umrahmten Ausdruckes für $\sqrt{2}$

$$= 1+\cfrac{1}{1+1+\cfrac{1}{1+1+\cfrac{1}{1+1+\cfrac{1}{1+1+\cfrac{1}{1+1+\cfrac{1}{1+1+\cfrac{1}{1+\sqrt{2}}}}}}}} = 1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{1+\sqrt{2}}}}}}}}$$

$$\cong 1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2}}}}}}} = 1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{\cfrac{5}{2}}}}}}} = 1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{2}{5}}}}}} = 1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{\cfrac{12}{5}}}}}}$$

$$= 1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{5}{12}}}}} = 1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{\cfrac{29}{12}}}}} = 1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{12}{29}}}} = 1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{\cfrac{70}{29}}}}$$

$$= 1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{29}{70}}} = 1+\cfrac{1}{2+\cfrac{1}{\cfrac{169}{70}}} = 1+\cfrac{1}{2+\cfrac{70}{169}} = 1+\cfrac{1}{\cfrac{408}{169}} = 1+\cfrac{169}{408} \cong \underline{1{,}414216}$$